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Piezo-Viscous Dependency and Couple Stresses on Performance of Rough Circular Plates

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Abstract: Purpose: This study examines the synergistic effects of pair stresses and piezo-viscous behavior on the performance attributes of squeeze film bearings featuring rough circular plates. The article analyzes the impact of the bearing surfaces and the porosity of one bearing surface on the pressure distribution and load-carrying capacity of the squeeze film bearing.

Design / Methodology / Approach: Modified Reynolds equations for a one-dimensional model are developed to account for both radial and azimuthal patterns of surface roughness. The results are then tabulated and statistically analyzed, with comparisons presented in terms of percentage differences.

Findings: The results show that radial roughness patterns (RRP) reduce the squeeze film approach time, load-carrying capacity, and pressure distribution.

Originality / Value: On the other hand, these performance measurements are enhanced by azimuthal roughness patterns, highlighting the crucial role that the orientation of roughness plays in squeeze film behavior.

Taxation and Keywords:
Reynolds' expression (RE), Deformable rough surface (DRS), Circular plates (CP), Couple Stresses (CS), Load bearing capacity (LBC), Squeezing response time (SRT).

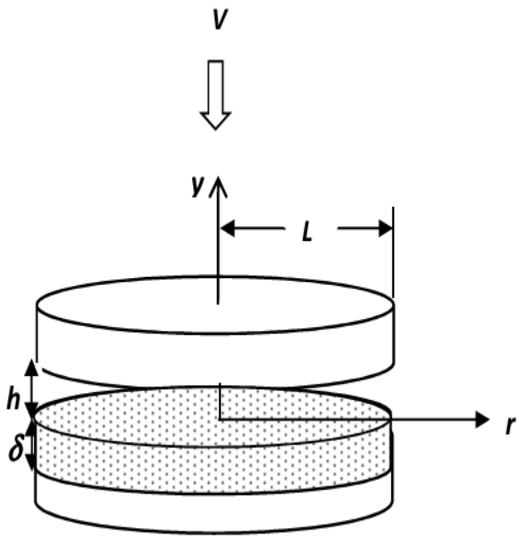
Subject Classification: 74A55 Theories of friction (tribology); 74Dxx Materials of strain-rate type and history type, other materials with memory (including elastic materials with viscous damping, various viscoelastic materials); 00A69 General applied mathematics {For physics, see 00A79 and Sections 70 through 86} 00A71 Theory of mathematical modeling 00A72 General methods of simulation.

I. Introduction

In tribology, squeeze film lubrication in various porous bearings has been thoroughly examined. Morgan and Cameron [1], Rouleau [2], Wu [3], Cusano [4], Prakash and Vij [5], and Tian [6] have all significantly contributed to this domain. The experiments, primarily centered on Newtonian fluids as lubricants, concluded that porous materials with differing permeabilities could possess the capabilities of various bearing disks. Stokes expanded micro-continuum theory incorporates couple stresses, body couples, and non-symmetric stress tensors [7]. Numerous researchers have examined various couple stress factors, including rotor-bearing machines, externally pressurized circular step thrust bearings, finite journal bearings, and the interaction between a sphere and a flat plate, as investigated by Lin [8–11]. Elsharkawy [12] examined the impact of misalignment on the performance of finite journal bearings lubricated with low-stress fluids in recent studies. In recent years, several thin film bearing models have been developed that incorporate the effects of couple stress and pressure-dependent viscosity (PDV). This encompasses research on circular step plates by Hanumagowda [13], the interaction between a sphere and a rough flat plate by Naduvinaman et al. [14], and the flow of a long cylinder and infinite plates by Lin and Lu [15]. While these studies offered significant insights into the impact of PDV on thin film bearings, they primarily focused on non-porous bearings. Lin et al. [16] Recently, Lin et al. [16] examined the influence of pressure-dependent viscosity in parallel circular plates coated with non-Newtonian fluids. Manjunatha Gudekote et al. [18] examined the influence of slip and inclination on the peristaltic transport of Casson fluid within an elastic tube featuring porous walls. Hassan et al. [19] examined the impact of porous twisted plate inserts on heat transfer efficiency and flow dynamics in fire tube boilers. Rajashekhar Choudhari et al. [20] examined the influence of hematocrit, slip, and total protein minus albumin (TPMA) on blood flow within an axisymmetric porous tube. This research examines the synergistic effects of piezo-viscous dependence and non-Newtonian couple stress on the squeeze film characteristics of porous circular plates. The findings indicate that as the permeability parameter approaches zero, the squeeze film behavior aligns with that of a non-porous bearing, corroborating Lins' results [16]. The review by Lin validates the precision of the existing model. Numerous studies have examined the surface roughness properties of different bearing types, including those by Tzeng and Saibel [31], Christensen and Tonder [32-34], Neuringer and Rosensweig [35], and Deheri et al. [37-38]. Consequently, it has become essential to examine and assess the synergistic impacts of piezo-viscous dependence and non-Newtonian couple stress on porous, deformable, rough circular plates.

II. Material and Methods

The lubrication assumptions include constant viscosity, laminar flow, no external forces, small lubricant film thickness, small fluid inertia, homogeneous porous area, Darcys law flow, continuous pressure, particle generation, and negligible heat generation. The research examines the effects of pressure-dependent viscosity on the success of porous circular plates made by a non-Newtonian couple stress fluid, focusing on the plates configuration and deformable roughness. $v = \frac{dh}{dt}$.



Geometry of porous parallel circular plates, V. K. Rajendrappa et. al. 2018

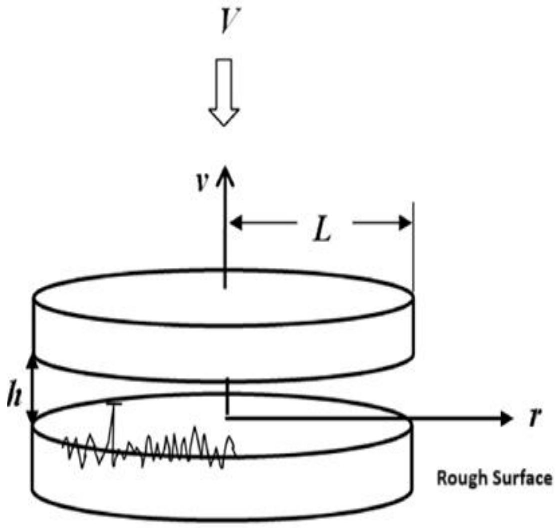


Diagram of squeeze film between rough circular plates, U. P. Singh, 2019

Consider the following variables in the context of circular plates: p represents the pressure between the plates, H denotes the thickness of the film between the plates, t is the squeeze film time, r indicates the lubricants viscosity, u stands for the velocity components, and is the material constant accountable for couple stresses. In addition, a uniform transverse magnetic field has been created between the two circular plates, which are oriented along the Z axis.

The basic equations for motion are specified by

$$\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}(\mathrm{ur}) + \frac{\partial \mathrm{v}}{\partial \mathrm{y}} = 0 \tag{1}$$

$$\mu\frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2} - \eta\frac{\partial^4 \mathrm{u}}{\partial \mathrm{y}^4} = \frac{\partial \mathrm{p}}{\partial \mathrm{r}} \tag{2}$$

$$\frac{\partial \mathrm{p}}{\partial \mathrm{y}} = 0 \tag{3}$$

In the equations, the symbols u and v denote the velocity components along the coordinate axes, μ represents viscosity, p signifies pressure, and η indicates the constant associated with couple stress fluid. The suitable boundary conditions for the velocity components u(r, y) and v(r, y) at the lower and upper surfaces are articulated as follows:
At the upper surface y = h:

$$u = 0,\ v = -v^{*} \text{ and } \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = 0 \tag{4a}$$

$$u = 0,\ v = -\frac{\partial h}{\partial t} \text{ and } \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = h \tag{4b}$$

The velocity components in the porous matrix are expressed as

$$u^* = \frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial r} \tag{5}$$

$$v^* = \frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial y} \tag{6}$$

Solving Equation (2) with conditions 4(a) and 4(b) we get

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial r}\left[y^2 - hy + 2l^2\left\{1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right\}\right] \tag{7}$$

Substituting the expression for u in the continuity equation (1) and integrating using the conditions 4(a), 4(b), (5) and (6), one gets the Reynolds' type presentation for smooth surface as:

$$\frac{\partial}{\partial r}\left[\frac{r}{\mu}\left\{h^3 - 12l^2h + 24l^3\tanh\left(\frac{h}{2l}\right) + \frac{12\phi k}{(1-\beta)}\right\}\frac{\partial p}{\partial r}\right] = 12r\frac{dh}{dt} \tag{8}$$

Flow into the permeable medium obeys Darcy's modified form of rule, whereas, hydro magnetic lubrication assumption holds in the film region. Tzeng and Saibel (1967) considered a method for irregular surface and accepted the one-dimensional film thickness to be of the shape

$$h(x) = h^*(x) + h_s(x) \tag{R1}$$

where $h^*(x)$ is the mean film thickness, $h_s(x)$ is the random deviation from the mean film thickness, h(x) is regarded as a random variable whose probability density function is either a Gaussian normal distribution function or Beta distribution function given by

$$f(h_s) = 1.09375\mathrm{C}^{-7}\left[1 - h_s^{\ 2}C^{-2}\right]; -c \le h_s \le c \tag{R2}$$

$$f(h_s) = 0, \text{ elsewhere} \tag{R3}$$

Distribution function above cited in the mathematical form is applied to obtain different parameters of roughness, like mean α, standard deviation σ and parameter ε (measure of symmetry) as

$$\alpha = E\,(h_s) \tag{R4}$$

$$\sigma^2 = E\,[(h_s - \alpha)^2] \tag{R5}$$

and

$$\varepsilon = E\,[(h_s - \alpha)^3] \tag{R6}$$

where

E is the expectancy operator

$$E(R) = \int_{-c}^{c} Rf(h_s)dh_s \tag{R7}$$

Under hydro-magnetic lubrication theory, stochastically averaging and adopting the properties of magnetic fluid lubrication (Neuringer and Rosensweig (1964), Bhat (2003), Christensen and Tonder (1970), Deheri et al. (2005), Rao and Prasad (2004)), the generalized Reynolds' equation is comes out as

$$\frac{\partial}{\partial r}\left[\frac{r}{\mu}\left\{a(h) - 12l^2[a(h)]^{1/3} + 24l^3\tanh\left(\frac{[a(h)]^{1/3}}{2l}\right) + \frac{12\phi k}{(1-\beta)}\right\}\frac{\partial p}{\partial r}\right] = 12r\frac{dh}{dt} \tag{R8}$$

Here

$$a(h) = (h + p_a p'\delta)^3 + 3(\sigma^2 + \alpha^2)(h + p_a p'\delta) + 3(h + p_a p'\delta)^2\alpha + 3\sigma^2\alpha + \alpha^3 + \varepsilon$$

$$\frac{\partial}{\partial r}\left[\frac{r}{\mu}\left\{b(h)-12l^2[b(h)]^{1/3}+24l^3\tanh\left(\frac{[b(h)]^{1/3}}{2l}\right)+\frac{12\phi k}{(1-\beta)}\right\}\frac{\partial p}{\partial r}\right]=12r\frac{dh}{dt} \quad \text{(LSR1)}$$

where

$b(h) = (h + p_a p'\delta)^{-3}[1 - \alpha (h + p_a p'\delta)^{-1/3} + 6(h + p_a p'\delta)^{-2}(\sigma^2 + \alpha^2) - 10(h + p_a p'\delta)^{-3}\{\varepsilon + 3\sigma^2\alpha + \varepsilon\}]$

The pressure dependent viscosity relation was analysed by Barus [17] and the expression is obtained as

$$\mu = \mu_0\, e^{(\alpha p)} \quad (9)$$

Substituting Equation (9) into Equation (8), the modified Reynolds' equation governing pressure dependent viscosity for porous circular plates lubricated with non-Newtonian fluid is expressed as

$$\frac{\partial}{\partial r}\left[f(h,m,\alpha,p)\frac{\partial p}{\partial x}\right]=12r\mu_0\frac{dh}{dt} \quad (10)$$

where

$$f(h,m,\alpha,p)=a(h)e^{-\alpha p}-12m^2[a(h)]^{1/3}e^{-2\alpha p}+24m^3e^{-\frac{5}{2}\alpha p}\tanh\left(\frac{a(h)e^{-0.5\alpha p}}{2m}\right)+\frac{12\phi k e^{-\alpha p}}{(1-\beta)}$$

and

$$l=\left(\frac{\eta}{\mu}\right)^{0.5}e^{-0.5\alpha p}$$

For convenience, let us introduce the following non-dimensional scheme.

$$\sigma^*=\frac{\sigma}{h_0} \qquad \alpha^*=\frac{\alpha}{h_0} \qquad \varepsilon^*=\frac{\varepsilon}{h_0{}^3}$$

$$\delta^*=\frac{\delta}{h_0} \qquad r^*=\frac{r}{L} \qquad l^*=\frac{l}{h_0}$$

$$p^*=\frac{ph_0{}^3}{\mu_0L^2\left(-\frac{dh}{dt}\right)} \qquad G=\frac{\alpha\mu_0L^2\left(-\frac{dh}{dt}\right)}{h_0{}^3} \qquad \psi=\frac{k\phi}{h_0{}^3}$$

The modified stochastically averaged Reynolds' type expression in dimensionless can be expressed as

$$\frac{\partial}{\partial r^*}\left[f(h^*,l^*,G,p^*,\psi)r^*\frac{\partial p^*}{\partial r^*}\right]=-12r^* \quad (11)$$

where

$$f(h^*,l^*,G,p^*,\psi)=A^*e^{-Gp^*}-12\left(l^{*2}\right)e^{-2Gp^*}\left(A^*\right)^{1/3}+24\left(l^{*3}\right)e^{-2.5Gp^*}\tanh\left(\frac{e^{0.5Gp^*}\left(A^*\right)^{1/3}}{2l^*}\right)+\frac{12\psi e^{Gp^*}}{(1-\beta)} \quad (12)$$

Here

$A^* = (1 + p^*\delta^*)^3 + 3(1 + p^*\delta^*)^2\alpha^* + 3(1 + p^*\delta^*)(\sigma^{*2} + \alpha^{*2}) + 3\sigma^{*2}\alpha^* + \alpha^{*3} + \varepsilon^*$

$$f(h^*,l^*,G,p^*,\psi)=B^*e^{-Gp^*}-12\left(l^{*2}\right)e^{-2Gp^*}\left(B^*\right)^{1/3}+24\left(l^{*3}\right)e^{-2.5Gp^*}\tanh\left(\frac{e^{0.5Gp^*}\left(B^*\right)^{1/3}}{2l^*}\right)+\frac{12\psi e^{Gp^*}}{(1-\beta)} \quad \text{(LSR2)}$$

where

$B^* = [(1 + p^*\delta^*)^{-3} - 3(1 + p^*\delta^*)^{-2}\alpha^* + 6(\sigma^{*2} + \alpha^{*2})(1 + p^*\delta^*)^{-1} - 10(\varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3})]$

The non-dimensional Reynolds' Equation (12) is observed to be highly non-linear and hence the small perturbation is adopted to find the analytical solution for the film pressure with the consideration of small values of the viscosity parameter0 $0 \le G \le 1$.The expression for p^* as follows.

$$p^* = p_0^* + Gp_1 \tag{13}$$

Substituting equation (13) into Reynolds Equation (11) and neglecting second and higher order terms of G. The following two equations responsible for pressure p_0^* and p_1^* are obtained as follows.

$$\frac{\partial}{\partial r^*}\left[r^*\frac{dp_0^*}{dr^*}\right] = -\frac{12r^*}{f_0^*\left(h^*,l^*,\psi\right)} \tag{14}$$

$$\frac{\partial}{\partial r^*}\left[r^*\frac{dp_1^*}{dr^*}\right] = -\frac{f_1^*\left(h^*,l^*,\psi\right)}{f_0^*\left(h^*,l^*,\psi\right)}\frac{d}{dr^*}\left\{p_0^* r^*\frac{dp_0^*}{dr^*}\right\} \tag{15}$$

Where,

$$f_0^*(h^*,l^*,\psi) = A^* - 12\left(l^{*2}\right)\left(A^*\right)^{1/3} + 24\left(l^{*3}\right)\tanh\left(\frac{\left(A^*\right)^{1/3}}{2l^*}\right) + \frac{12\psi}{(1-\beta)} \tag{16}$$

$$f_1^*(h^*,l^*,\psi) = -A^* + 6\left(l^{*2}\right)\left(A^*\right)^{1/3}\left[4 + \sec h^2\left(\frac{\left(A^*\right)^{1/3}}{2l^*}\right)\right] - 60\left(l^{*3}\right)\tanh\left(\frac{\left(A^*\right)^{1/3}}{2l^*}\right) + \frac{12\psi}{(1-\beta)} \tag{17}$$

The solution for p_0^* and p_1^* are obtained by solving Equation (14) and (15) as

$$p_0^* = \frac{3(1-r^{*2})}{f_0^*\left(h^*,l^*,\psi\right)} \tag{18}$$

$$p_1^* = -4.5\frac{f_1^*\left(h^*,l^*,\psi\right)(1-r^{*2})^2}{f_0^{*3}\left(h^*,l^*,\psi\right)} \tag{19}$$

Substituting these expressions into Equation (13) the dimensionless pressure is obtained as

$$p^* = \frac{3\left(1-r^{*2}\right)}{f_0^*\left(h^*,l^*,\psi\right)} - 4.5G\frac{f_1^*\left(h^*,l^*,\psi\right)\left(1-r^{*2}\right)^2}{f_0^{*3}\left(h^*,l^*,\psi\right)} \tag{20}$$

The load W^* on the bearing can be obtained by

$$W^* = \frac{Wh_0^{\,3}}{\mu_0 L^4\left(-\frac{dh}{dt}\right)} = 2\pi\int_0^1 p^* r^* dr^* \tag{21}$$

After performing the integration, the load on the bearing in dimensionless form can be obtained as

$$W^* = 4.7123\left\{\frac{f_0^{*2}\left(h^*,l^*,\psi\right) - Gf_1^*\left(h^*,l^*,\psi\right)}{f_0^{*3}\left(h^*,l^*,\psi\right)}\right\} \tag{22}$$

The squeeze film time t can be obtained by integrating (22) with respect to h^* under the condition $h^* = 1$ at $t = 0$ as follows.

$$T^* = \frac{Wth_0^{\,2}}{\mu_0 L^4} = 4.7123\int_{h_1^*}^{1}\left\{\frac{f_0^{*2}\left(h^*,l^*,\psi\right) - Gf_1^*\left(h^*,l^*,\psi\right)}{f_0^{*3}\left(h^*,l^*,\psi\right)}\right\}dh^* \tag{23}$$

III. Results

The report looks at the effects of piezo-viscosity and a few others on porous circular plates characteristic. It employs Barus model, Stokes continuum theory, and the Morgan–Cameron approximation. According to the findings, incorporating a few stresses, local elastic deformation, and ambient pressure will increase the bearing systems efficiency. Surface roughness, as well as standard deviations and influence performance. The research additionally examines pressure-dependent viscosity as the permeability parameter nears zero; As the permeability parameter diminishes, the squeeze film characteristics identified in this study converge with those of Lin [16], as illustrated in Table 1. The findings were consistent with Lin's review; in comparison to the conventional bearing case, an increase in pressure, load-carrying capacity, and squeeze film duration led to a reduction in tension, load-carrying capacity, and squeeze film duration.

Data Availability Declaration of Competing Interest: There is up to certain extent data has been used for this research and cited at the proper place/s. Author/s declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

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IV. Discussion and Conclusion

Pictorial Results and Discussions

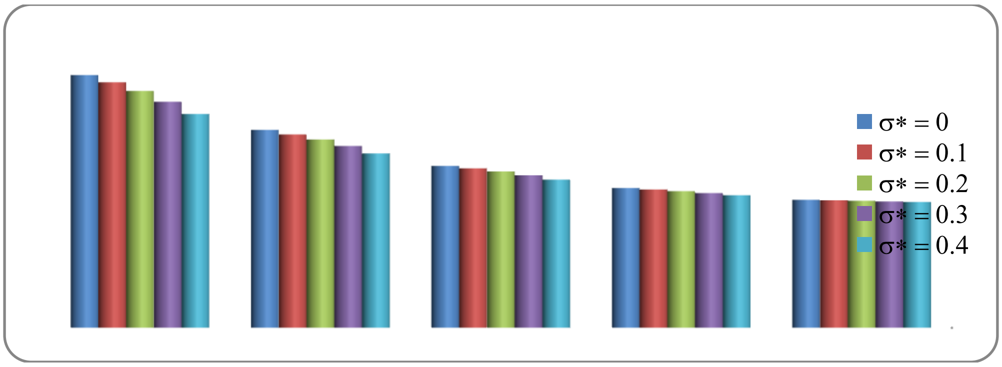

Figure: 1 Profile of Load with regards to Variance

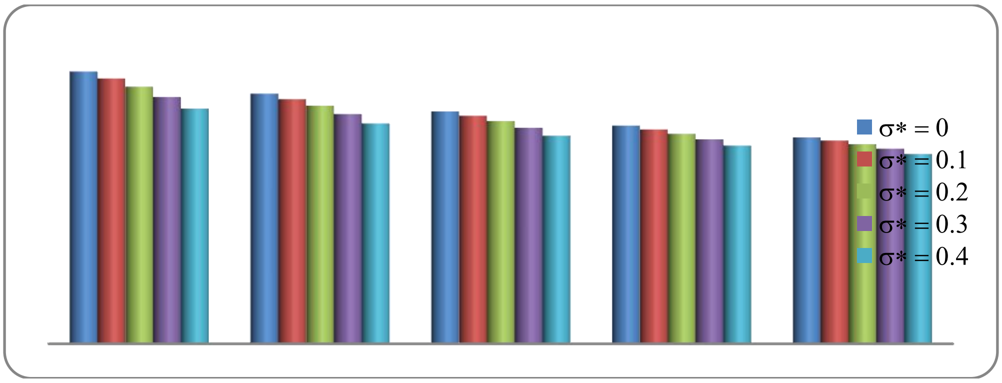

Figure: 2 Load Carrying Capacity w. r. t. Skewness

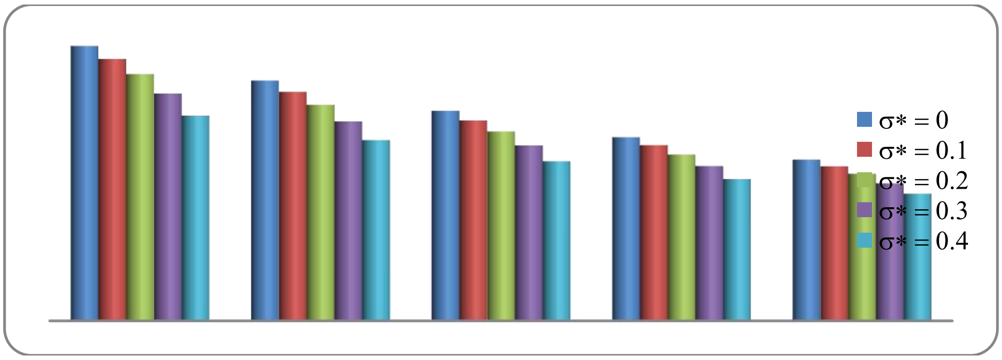

Figure: 3 Load Bearing Capacity w. r. t. Deformation

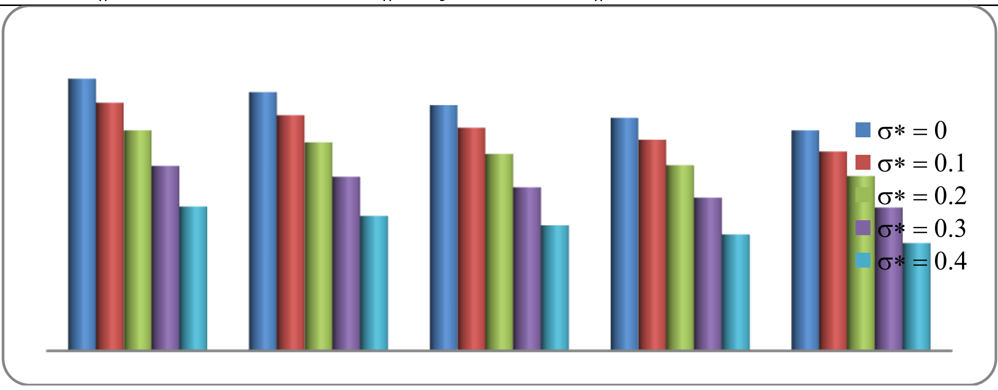

Figure: 4 Distribution of Load w. r. t. Ambient Pressure

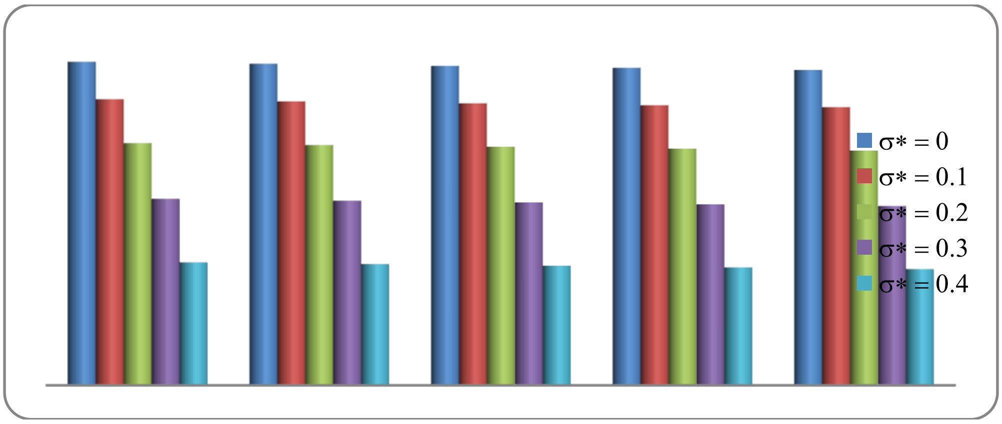

Figure: 5 Load Bearing Capacity w. r. t. Porosity

Tabular Presentation

1	α*=0	α*=0.1	α*=0.2	α*=0.3	α*=0.4
σ* = 0	0.977855511	0.89504489	0.83217001	0.770860344	0.701029805
σ* = 0.1	0.994286403	0.900380091	0.82903625	0.761348696	0.687644173
σ* = 0.2	1.015408879	0.907119919	0.825146991	0.749763533	0.67169927
σ* = 0.3	1.046107603	0.916689428	0.819753823	0.734100469	0.650805184
σ* = 0.4	1.087599012	0.929225914	0.812907729	0.714890165	0.62631194

2	ε*=0	ε*=0.1	ε*=0.2	ε*=0.3	ε*=0.4
σ* = 0	0.70523366	0.564094961	0.521435812	0.636396086	1.68980112
σ* = 0.1	0.74554536	0.584331951	0.521152421	0.591374074	1.208026466
σ* = 0.2	0.802575546	0.614835454	0.52734734	0.554499587	0.910904319
σ* = 0.3	0.896226163	0.667592292	0.546811225	0.527555652	0.70569339
σ* = 0.4	1.044428856	0.75413743	0.588810054	0.5216756	0.584228232

3	δ*=0	δ*=0.05	δ*=0.1	δ*=0.15	δ*=0.2
σ* = 0	11.84144817	10.80825879	9.964079808	9.263441066	8.67403365
σ* = 0.1	14.29003604	12.93306198	11.83687888	10.93587673	10.18423465
σ* = 0.2	18.49210033	16.52089435	14.95604598	13.68865823	12.64463635
σ* = 0.3	28.04873042	24.45058177	21.68727419	19.5101987	17.75821155
σ* = 0.4	55.43251386	45.86956331	39.02038493	33.92003865	30.00199931

4	P*=0	P*=0.1	P*=0.2	P*=0.3	p*=0.4
σ* = 0	11.84144817	10.80825879	9.964079808	9.263441066	8.67403365
σ* = 0.1	14.29003604	12.93306198	11.83687888	10.93587673	10.18423465
σ* = 0.2	18.49210033	16.52089435	14.95604598	13.68865823	12.64463635
σ* = 0.3	28.04873042	24.45058177	21.68727419	19.5101987	17.75821155
σ* = 0.4	55.43251386	45.86956331	39.02038493	33.92003865	30.00199931

5	ψ=0.001	ψ=0.003	ψ=0.005	ψ=0.007	ψ=0.009
σ* = 0	9.964079808	9.952392714	9.94072617	9.929080128	9.91745454
σ* = 0.1	11.83687888	11.8218802	11.80691	11.79196823	11.77705479
σ* = 0.2	14.95604598	14.93497351	14.91394554	14.89296195	14.87202262
σ* = 0.3	21.68727419	21.65101711	21.61485088	21.5787752	21.54278975
σ* = 0.4	39.02038493	38.93447366	38.84884581	38.76350013	38.67843539

Nomenclature:

RRP Radial Roughness Pattern (mm)

PD Pressure Distribution: Pressure is defined as the ratio of a force divided by the area over which that force acts; thus, the units of pressure, force per unit area, are newtons per square meter (=1 Pa), dynes per square centimeter ($=10^{-6}$ bars), or pounds per square inch (at sea level 1 atm pressure = 1.0133×10^5 Pa = 1.0133 bars = 14.696 lb

LCC Load Carrying Capacity: Load bearing capacity of the supports in tons per linear meter of the face (20 t ≤ PM ≤ 260 t).

SF Squeeze Film:

ARP Azimuthal Roughness Pattern: ARS ... Rms roughness, [nm], σ. Spatial frequency, [μm^{-1}], f. Specular reflectance ...

MHD Magnetohydrodynamic

PVD Piezo-Viscous Dependency

NNCS Non-Newtonian Couple Stresses

DRS Deformable Rough Surfaces (mm)

ϕ Permeability of porous facing: The common unit is the darcy (D) [about 10^{-12} m^2];

H_0 Minimum film thickness of porous facing (m)

σ Standard deviation (Nm / Cm / Meter)

ε Skewness: Skewness has no units: it's a pure number, like a z-score.

α Variance always has squared units.

δ Deformation is the change in the shape or size of an object. It has dimension of length with SI unit of meter (m).

σ^* Non-dimensional standard deviation: σ / h_0

ε^* Non-dimensional skewness: ε / h_0^3

α^* Non-dimensional variance: α / h_0

δ^* Non-dimensional deformation: δ / h_0

ψ $= \frac{12\emptyset H}{h_0^3}$The porosity is conventionally given the symbol φ, and is expressed either as a fraction varying between 0 and 1, or a percentage varying between 0% and 100%. Sometimes porosity is expressed in 'porosity units', which are the same as percent (i.e., 100 porosity units (pu) = 100%).

References

[1]. Morgan, V. T., and A. Cameron. "Mechanism of lubrication in porous metal bearings." In Proceedings of the Conference on Lubrication and Wear, Institution of Mechanical Engineers, London, pp. 151-157. 1957.

[2]. Rouleau, W. T. "Hydrodynamic lubrication of narrow press-fitted porous metal bearings." Journal of Basic Engineering 85, no. 1 (1963): 123-128.

[3]. Wu, Hai. "Squeeze-film behavior for porous annular disks." Journal of Lubrication Technology 92, no. 4 (1970): 593 596.

[4]. Cusano, C. "Lubrication of porous journal bearings." Journal of Lubrication Technology 94, no. 1 (1972): 69-73.

[5]. Prakash, J., and S. K. Vij. "Hydrodynamic lubrication of a porous slider." Journal of Mechanical Engineering Science 15, no. 3 (1973): 232-234.

[6]. Tian, Yong. "Static study of the porous bearings by the simplified finite element analysis." Wear 218, no. 2 (1998): 203-209.

[7]. Stokes, Vijay Kumar. "Couple stresses in fluids." The physics of fluids 9, no. 9 (1966): 1709-1715.

[8]. Lin, Jaw-Ren. "Effects of couple stresses on the lubrication of finite journal bearings." Wear 206, no. 1-2 (1997): 171-178.

[9]. Lin, Jaw-Ren. "Static and dynamic characteristics of externally pressurized circular step thrust bearings lubricated with couple stress fluids." Tribology International 32, no. 4 (1999): 207-216.

[10]. Lin, Jaw-Ren. "Squeeze film characteristics between a sphere and a flat plate: couple stress fluid model." Computers & Structures 75, no. 1 (2000): 73-80.

[11]. Lin, Jaw-Ren. "Linear stability analysis of rotor-bearing system: couple stress fluid model." Computers & Structures 79, no. 8 (2001): 801-809.

[12]. Elsharkawy, Abdallah A. "Effects of misalignment on the performance of finite journal bearings lubricated with couple stress fluids." International journal of computer applications in technology 21, no. 3 (2004): 137-146.

[13]. Hanumagowda, B. N. "Combined effect of pressure-dependent viscosity and couple stress on squeeze-film lubrication between circular step plates." Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology 229, no. 9 (2015): 1056-1064.

[14]. Naduvinamani, N. B., A. Siddangouda, G. Hiremath Ayyappa, and S. N. Biradar. "Effects of pressure-dependent viscosity variation on the squeeze film lubrication between a sphere and a rough flat plate with couple stress fluids." Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology 229, no. 9 (2015): 1114-1124.

[15]. Lin, Jaw-Ren, and Rong-Fang Lu. "A closed-form solution in piezo-viscous squeeze films between a long cylinder and an infinite plate." Industrial Lubrication and Tribology 66, no. 3 (2014): 505-508.

[16]. Lin, Jaw-Ren, Li-Ming Chu, and Long-Jin Liang. "Effects of viscosity-pressure dependency on the non-Newtonian squeeze film of parallel circular plates." Lubrication science 25, no. 1 (2013): 1-9.

[17]. Barus, Carl. "ART. X.--Isothermals, Isopiestics and Isometrics relative to Viscosity." American Journal of Science (1880-1910) 45, no. 266 (1893): 87.

[18]. Manjunatha, G., and Rajshekhar V. Choudhary. "Slip effects on peristaltic transport of Casson fluid in an inclined elastic tube with porous walls." Journal of Advanced Research in Fluid Mechanics and Thermal Sciences 43, no. 1 (2018): 67-80.

[19]. Hassan, S., M. K. Roslimb, and R. M. Zainc. "Simulation Studies on the Effect of Porous Twisted Plate Inserts on the Performance of Fire Tube Steam Packaged Boiler."

[20]. Choudhari, Rajashekhar, Manjunatha Gudekote, and Naveen Choudhari. "Analytical Solutions on the Flow of blood with the Effects of Hematocrit, Slip and TPMA in a porous tube." Journal of Advanced Research in Fluid Mechanics and Thermal Sciences 47, no. 1 (2018): 201-208.

[21]. Neminath Bhujappa Naduvinamani, Siddangouda Apparao, Hiremath Ayyapa Gundayya and Shivraj Nagshetty Biradar, 2015, Effect of Pressure Dependent Viscosity on couple stress squeeze film lubrication between Rough parallel plates, Tribology Online, 10(1) pp.76-83.

[22]. Hanummagowda, Shivakumar, Raju, Santhosh Kumar, Combined effect of pressure dependency viscosity and micro polar fluid on squeeze film circular stepped plates IJMTT, 37(3), 2016, 175-183.

[23]. Noor Jahan, B N Hanumagowda, A Salma and H M Shivakumar, Combined effect of piezo-viscous dependency and Couple Stresses on Squeeze-Film Characteristics of Rough Circular Plates, National Conference on Mathematical Techniques and its Applications (NCMTA 18) IOP Publishing IOP Conf. Series: Journal of Physics: Conf. Series 1000 (2018) 012082 doi :10.1088/1742-6596/1000/1/012082.

[24]. Nallusamy, Synthesis and performance characterization of nano lubricants proposed for heavy load ball bearing, Journal of Nano Research, 2018, 54, pp. 75–87.

[25]. Karthikeyan, Experimental investigation of wear properties on aluminium-6061 based silicon carbide composites using SEM, International Journal of Mechanical Engineering and Technology, 2018, 9(10), pp. 454–463

[26]. Prabu, N. M., Influence of boron nitride nano additives in cutting fluid for improving surface roughness with MRR, International Journal of Nanomanufacturing, 2020, 16(1), pp. 61–75

[27]. Prin. Dr. Ms. Pragna A. Vadher, Dr. Dhinesha Ruwanthi Perera, Dr. Sanjeev Kumar, Dr. Gunamani B. Deheri, Rakesh M. Patel, A study of deformable rough surfaces on finite slider bearing, Technische Sicherheit (Technical Security) Technische Sicherheit ISSN:2191-0073, ISSN:1434-9728/E-ISSN:1436-4948 || Impact Factor 6.1, 2025, 25(4), pp. 12-23.

[28]. Rakesh M. Patel, Prin. Dr. Pragna A. Vadher, Dr. Gunamani B. Deheri, Dr. Bharatkumar N. Valani, Ferrofluid-Based Tilted Deformable Rough Porous Pad Bearing, GIS JOURNAL OF SCIENCE, 2025, 12(2), pp. 348-359.

[29]. Dr. G. M. Deheri, Dr. S. G. Sorathiya, Dr. J. V. Adeshara, Dr. H. P. Patel and Rakesh M. Patel, Effect of Slip Velocity on Longitudinal Rough Hydro magnetic Squeeze Film Conducting Rotating Circular Plates, Journal of Materials and Engineering, 2025, 3(2), pp. 137-143.

[30]. Prin. Dr. Pragna A. Vadher, Dr. Gunamani B. Deheri, Dr. Bharatkumar N. Valani and Rakesh M. Patel, Behaviour of Deformable Elliptical Plates with Micropolar Fluids, Journal of Technology, 2025, 12(12), p.p. 693-703.

[31]. Tzeng, S. T. and Saibel, E., Surface roughness effect on slider bearing lubrication, Trans. ASME, J. Lub. Tech., 10, p.p. 334-338, 1967.

[32]. Christensen, H. and Tonder, K. C., The hydrodynamic lubrication of rough bearing surface of finite width, ASME-ASLE Lubrication Conference 1970; Paper No.70- Lub-7, 1970.

[33]. Christensen, H. and Tonder, K. C., Tribology of Rough Surfaces, Parametric Study and Comparison of Lubrication Models, SINTEF Report, no. 22/69 18, 1969a.

[34]. Christensen, H. and Tonder, K. C., Tribology of Rough Surfaces, Stochastic Models of Hydrodynamic Lubrication, SINTEF Report, no. 10/69–18, 1969b.

[35]. Neuringer,J. L., Rosensweig, R. E., Magnetic fluid, Physics of fluids, Vol. 12(7), 1964.

[36]. Bhat, M. V., Lubrication with a magnetic fluid. Team Spirit (India) Pvt. Ltd., New Delhi, 2003.

[37]. Patel R.M. and Deheri G.M., Magnetic fluid based squeeze film behaviour between rotating porous circular plates with a concentric circular pocket and surface roughness, effects, Int. J. of Applied Mechanics and Engineering, 8(2), 271-277, 2003.

[38]. Vadher, P., Daheri, G. M., and Patel, R. M., Hydrpmagnetic squeeze film between conducting porous transversely rough triangular plates, Journal of Engineering Annals of Faculty of engineering Hunedora, 6(1), 155-168, 2008.